Quick Way to Choose the Type of Pans

Contents

1 Introduction

In this problem, we are asked to improve the efficiency of baking process, namely, the maximum sum area of cakes (in a certain thickness) once can be placed in an oven and the extent to which the cakes are evenly baked, through the adjustment of the shape of pans. Therefore, it involves both thermodynamics theory and two-dimensional packing problem.

In the field of thermodynamics, the heat transfer processes within an oven-like enclosure have been thoroughly discussed. And food industry has already used its results to simulate food baking process with computational fluid dynamics(CFD)[N. Chhanwal 2010]. However, their models don't fit well with our problems, because they excessively focused on the thermal filed in the oven, established by heat radiation (dominantly), convection and conductivity, and on the chemical changes of food during the heating process. On the contrary, emphases should be put on the container itself in this problem—primarily on its shape and heat distribution, which means that a pan, if regarded as an independent system, could be studied in a relatively simple thermal atmosphere in which its own thermal properties will be shown more clearly. Just like what it's universally accepted that the attribution of a system could be presented by its response to the ideal impulse, the thermal attribution of a pan could be, to some extent, presented by the heat distribution under ideal boundary condition. The ideal boundary condition here indicates that the edge of pan has a constant temperature.

On the other hand, the problem's mathematical essential is captured as a two dimensional packing problem, whose objective is waste minimisation. This is a well-know pallet loading problem. In current researches, it's commonly addressed by the use of schemes of dynamic programming, tabu search, genetic algorithm. M.Z. Arslanov[2000] in his paper proposed an algorithm of linear complexity based on the number-theoretical method. However, it restricts itself in identical rectangular packing problem, which is not properly to solve our polygon arrangement problem. A. Miguel[2006] has addressed irregular strip packing problem with a hybrid of heuristic algorithms. And Boris D., on the contrary, has used computational experiments to study a relatively simple situation—"dense packing of congruent circles in rectangular with a variable aspect ratio". However, in our problem, numbers of regular polygons or rectangles have to be packed into a rectangle whose parameter w/l , which represents the ratio of its width and length, is given. So we developed another model, in which a group of polygons are represented by overlapped circles. The circles cover each other in the proportion of $k\%$, which has a negative correlation with number of polygon's edges.

Our paper is organised as follows. First, an ideal thermal model is developed to simulate the heat distribution of pans in different shapes. Next, under the requirement of maximizing the number of pans, a group of overlapped circles is used to approximately calculate the number of pans that can be placed into an oven. Finally, according to the heat distribution computed by the first model, the pan with most even distribution is selected. And in order to combine the first condition with the second one, the weight p and $(1-p)$was assigned to adjust the priority putted on the two conditions. With weight coefficient p , the whole selecting function could be developed to meet different baking requirements in practise.

2 Analysis of the Problem

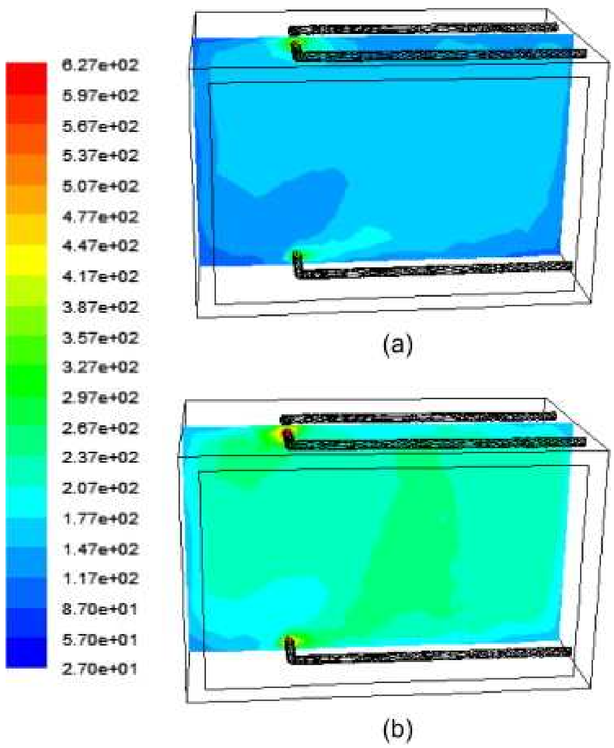


Figure 1: Temperature profile for baking oven without product (DO model) at (a) 180 s and (b) 1200 s.[N. Chhanwal 2010]

2.1 Assumption

- **The thermal field in an oven is ideal—temperature within the oven is constant.** Figure.1 shows the heat distribution simulated by N. Chhanwal in an running oven in different times[N. Chhanwal 2010]. In its central part, heat distribution are roughly the same, which is supportive for our ideal thermal atmosphere. Meanwhile, several factors such as shape and boundary condition, can contribute to the heat distribution of pans. As is mentioned in the introduction, our purpose is to explore the heat distribution of the pan's own in different shapes. Therefore, in order to focus on the impact of shapes, we simplify the boundary condition to the constant temperature one.

- **No interaction between pans in an oven.** As we focus on the influence exerted by the shape of pans on heat distribution and maximum number of pans, it's unnecessary to account for how a group of same pans interact with each other. It won't disclosure the properties of pan itself but of the group of pans. The interaction between a group of pans can lead to the increasing complexity of the boundary condition which on the one hand may decrease the effect of the shape, on the other hand have no direct impact on our purpose – to optimize the efficiency of baking process.

- **The heat distribution of pans is decided only by its shape and material under a certain boundary condition.** This assumption seems to be a conclusion from the two assumptions above. It stresses again the idealisation of the thermal atmosphere and that the research object is the individual pan itself rather than a group of pans or the oven.

- **Only convex regular polygons and rectangles are taken into consideration.** This assumption is for simplification and manufactural practice.

- **The rectangle can be only placed parallel to the width or length edge of an oven .** This assumption matches with the baking habits in daily life. No one likes to place the pans like a complicated collage.

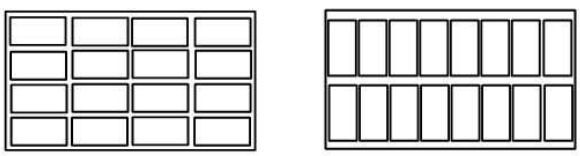


Figure 2: Two arrangement types of rectangular pan

2.2 Requirement One

Based on our assumption, the heat transfer process from the oven heat source to the pans could be abandoned. And the heat within the pan is transferred in the mechanism of thermal conductivity. Therefore according to the thermodynamic theory, a parabolic partial differential equation is used to compute the heat distribution within the pan,

$$\frac{\partial T}{\partial t} - \alpha(\frac{\partial^2 T}{\partial x} + \frac{\partial^2 T}{\partial y} + \frac{\partial^2 T}{\partial z}) = 0 \tag{1}$$

$$\alpha = -\frac{k}{\rho C_p} \tag{2}$$

In calculating process, the pan is simplified to a two-dimensional plane, as its uneven distribution of heat primarily results from the heat transferred from its sides, not from the bottom or the top. Consequently, the boundary condition for the equation above is a constant T_b on each sides. For every polygon, simulation time should be the same t_b . We can sampling in the process to see the gradual change at different time.

2.3 Requirement Two

As we captured the mathematical essential of the problem to be a two dimensional packing optimisation, heuristic algorithms are expected to be used to acquire more approximate results. However, our assumption send us to a relatively simple situation. In the book *mathematical modeling*[Sanxing Wu 1993], he addressed a covering problem in which a certain rectangular should be covered with minimum number of circles under the condition that each two circles should have a common area, and the ratio of common area to the area of circle must be no more than $k\%$(referred to as area coefficient). In this book, circles with area coefficient in certain range are replaced by regular polygon whose number of sides are corresponding to the are coefficient, Figure 3 shows the example of his model.

Enlightened by its method, we take the inverse process, namely simplify the regular polygons to group of circles overlapped with an area coefficient k, which, as stated above, is corresponding to the number of polygon's sides. However, considering whether 360°can be divided exactly by the degree of polygon's interior angle, further approximation has to be taken. Accurate value of k can be calculated only for triangle, square and hexagon, so in order to obtain an approximate relationship between N and k, by observing the scatter plot, we decide to use a exponential function like $N = Ae^{Bk}$ to fit the known data. We could transfer former function into a linear equation like $lnN = ak + b$, shown as below:

$$k = e^{-0.628n+0.496}. \tag{3}$$

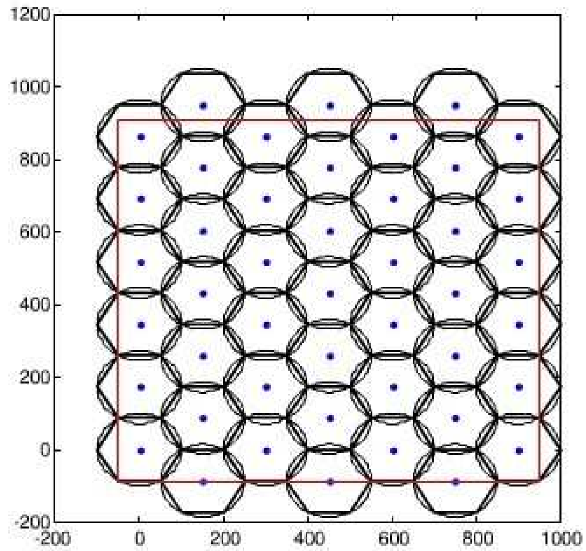


Figure 3: Circles with area coefficient $k \leq 5.7$ are treated as regular hexagon

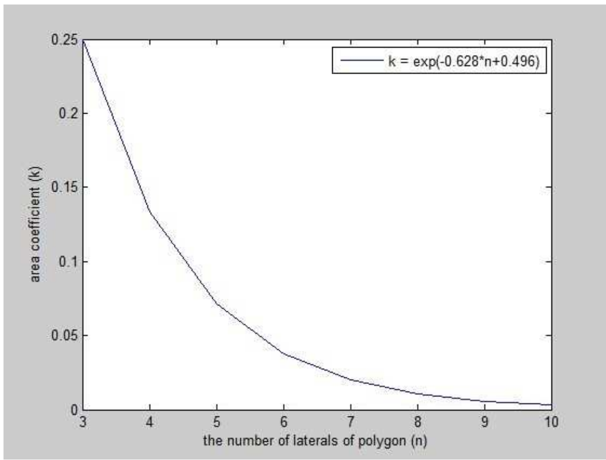


Figure 4: Linear fitting equation for n and k

In this linear regression case, coefficient of determination is 0.994, indicating that the result of linear fitting is acceptable.

Polygon	area coefficient k	$R_o/\sqrt{A}$	$R_i/\sqrt{A}$
3	39.09	0.88	0.44
4	18.15	0.71	0.5
5	7.11	0.65	0.53
6	5.77	0.62	0.54
7	2.02	0.60	0.55
8	1.08	0.59	0.55
$\vdots$	$\vdots$	$\vdots$	$\vdots$

The data acquired from the linear equation is listed in the table above, R_o presents the radius of inscribed circle, R_i presents the radius of circumcircle, now the problem is converted to the traditional circle packing problem. There are two basic densest arranging patterns shown in Figure.5, and the line between adjacent circles is either parallel to the width or to the length edge of the oven, then the approximate optimal solution could be acquired.

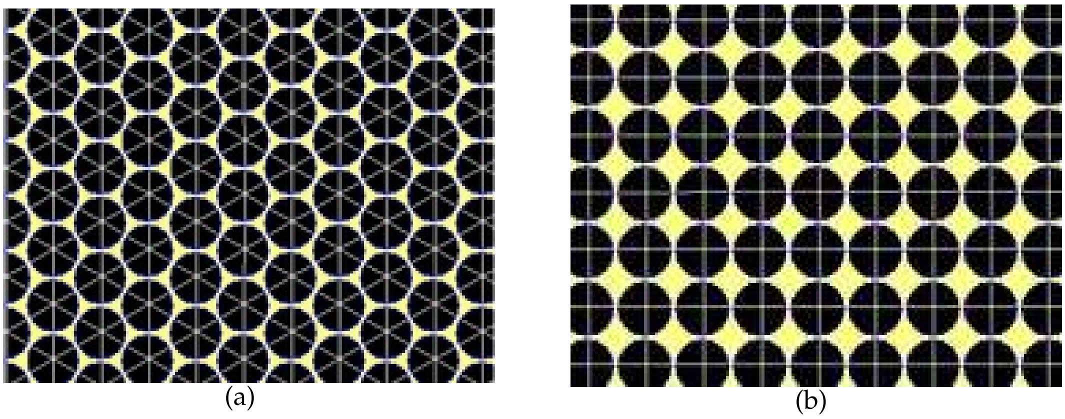


Figure 5: possible densest packing patterns.[http://en.wikipedia.org/wiki/Circle_packing]

So far, approximate optimal solution for regular polygons could be given according to the "overlapped circle" method. However, packing identical rectangle is a much more complicated and long-studied problem. Traditional algorithm tackling with this problem is dynamic programming, genetic algorithm and tabu search, which are not practical for our cooking problem due to its extremely heavy complexity . Here we apply the simplest mesh arrangement, namely, the rectangle can be only placed parallel to the width or length edge of an oven , to explore the maximum number of pans for each current size of oven, aiming at provide a practical optimal reference for the current facilities.

3 Calculating and Simulating of the Model

3.1 Requirement One

Heat distribution of our first model is calculated using finite difference method. Specifically, the heat equation for our two-dimensional, constant boundary model is given as:

$$\frac{\partial T}{\partial t} - \alpha(\frac{\partial^2 T}{\partial x} + \frac{\partial^2 T}{\partial y}) = 0 \tag{4}$$

$$boundary\ condition = T_{const} \tag{5}$$

then the parabolic partial equation should be discretized for computer simulation as:

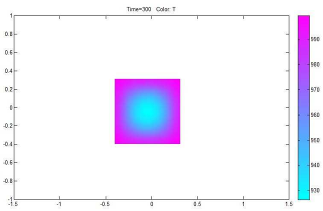
$$\frac{T_{i,j}^{k+2} - T}{\triangle t} - \lambda(\frac{T_{i,j+1}^{k+1} - 2T_{i,j}^{k+1} + T_{i,j-1}^{k+1}}{\triangle x^2} + \frac{T_{i+1,j}^{k} - 2T_{i,j}^{k} + T_{i-1,j}^{k}}{\triangle y^2}) = 0 \tag{6}$$

in which $\lambda = -\frac{k}{\rho C_p} = 3.8$, characteristic value of stainless steel. A coefficient β (referred as even coefficient)was proposed to measure the extent to which the heat was evenly distributed, which is defined as given equation:

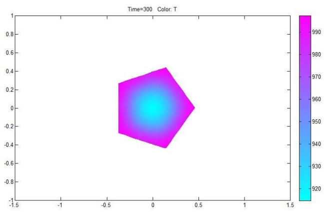
$$\beta = \frac{\bar{\sigma}}{r} \tag{7}$$

The even coefficient describes to what extent the outer edge of a bread is evenly cooked. $\bar{\sigma}$ presents the standard deviation of temperature of the pan, and $r = T_{max} - T_{min}$ presents the temperature range of the pan. Generally speaking, the standard deviation shows how much variation exists from the average, but the standard deviation of temperature of the pan can not reflect the situation on the outer edge. However, standard deviation, if divided by the range of temperature, could empirically eliminate the deviation resulted from the difference of distance from the boundary to the center.

Four polygons' heat distribution at certain time are acquired by using finite difference method, figures show as follows:



(a) 4



(b) 5

Meanwhile, the even coefficients are calculated and listed in the table below:

(c) 6 (d) circle

Figure 6: heat distribution for different shapes

Number of sides	even coefficient β	α
4	0.2813	0.7
5	0.2615	0.81
6	0.2737	0.87
7	0.2578	0.92
∞(circle)	0.2556	1

The conclusion is reached for the first condition that circle pans are most evenly heated.

It generally fit the rule that the more angles are there, the less evenly heat will be distributed, though there is an odd for hexagon. The table presents the simple, discrete relationship between number of sides and the even coefficient. The parameter $\alpha = \frac{R_i}{R_o}$, which presents the ratio of radius of inscribed circle to the radius of circumcircle, describes the degree of the circle.

3.2 Requirement Two

In the table below, $\frac{w_o}{l_o}$ presents the width to the length ratio for the oven, and $\frac{w_p}{l_p}$ presents the width to the length ratio for the pan, α is the radius of inscribed circle to radius of circumcircle of polygon ratio. For simplification and practice, we only explore the maximum number of rectangle under the size of widely-used ovens and pans. So we acquire a simple, discrete relationship between α and $N, W/L$.

$$N = max(\lfloor\frac{W_o}{W_p}\rfloor \bullet \lfloor\frac{L_o}{L_p}\rfloor, \lfloor\frac{W_o}{L_p}\rfloor \bullet \lfloor\frac{L_o}{W_p}\rfloor) \tag{8}$$

$\frac{w_o}{l_o} \backslash \frac{w_p}{l_p}/\alpha$	0.2667/0.2577	0.6364/ 0.5369	0.6923/0.5692
0.5	18	20	20
0.6054	36	36	36
0.7042	24	25	25
0.7936	48	50	56

$\frac{w_o}{l_o}\backslash\alpha$	0.70	0.81	0.87	0.92	0.93	1
0.5	18	18	18	18	20	18
0.61	35	28	35	28	28	28
0.70	21	20	21	20	20	20
0.79	49	42	44	40	40	40

These two tables are for the selection of polygon pans, for each size of oven, the maximum number of pans are listed in the table. One facing a certain oven, can easily choose the type of pans which can be displaced into an oven most.

Maximum number of pans and maximum even distribution of heat are two contradictory condition, and weights p and $(1-p)$ are assigned to select different type of pans to deal with different situation in which high baking rate or high quality is demanded. The parameter $\alpha = \frac{R_i}{R_o}$, which presents the ratio of radius of inscribed circle to the radius of circumcircle, describes the degree of circle, p and α are combined in the follow equation:

$$\alpha(p, W/L) = p\alpha_1(N, W/L) + (1-p)\alpha_2(\beta) \tag{9}$$

The weight p is used as a situation parameter, When p is close to 1, it indicates that the baker is in a hurry, or there are lots of customers in a breadstore, in which situation the high rate of baking is demanded. On the contrary, when p is close to 0, indicates the customers is a very picky one, who loves the even edge of bread and therefore, the even distribution of heat is demanded.

A certain situation indicates the value of p, man who have puzzles on which type of pans should be selected, just check the size of their oven, look through the two tables given in the second model,acquire the α_1, and $\alpha_2 = \alpha_{circle}$. Then the total α, which presents the type of pans,is calculated, and can be found through the correlation in the table.

4 Conclusions

We model heat transfer process by using two dimensional heat equation, and only heat transfer within the pan is considered. The pan is simplified to a two dimensional plane with constant temperature on its edges. As a result, the heat distribution over the outer edge of a pan is strongly influenced by the smooth degree of edge., which can be indicated by the parameter α. Through simulation, conclusion is reached that the more closer to circle it is, the more evenly the heat is distributed.

The arrangement of pans with in a oven is modeled a packing problem. In our model, a group of polygons are represented by overlapped circles. The circles cover each other in the proportion of $k\%$, which has a negative correlation with number of polygon's edges. A reference sheet has been achieved in our paper, which can be referred to select pans type under current size of pans and oven to deal with different demanding for baking rate and quality.

5 Evaluate of the Mode

In our model, heat distribution of the pan is analysed in an ideal thermal atmosphere. However, in reality, the thermal field in an oven is more complicated and the interaction between the pans can not be ignored. Pans in different place in the oven would have different heat distribution. And they may show some features which can be observed only when analysed as a group. However, group feature depends largely on the size of oven, so can not be used to select the type of pans.

In the terms of the definition of even distribution, we use the even coefficient $\beta = \frac{\bar{\sigma}}{r}$ to approximately estimate to what extent heat is evenly distributed. A more rigorous method to measure it should be in this way: A circle does exist in every heat distribution picture, whose form follows the thermal conductivity. If a square pan is evenly baked on its outer edge, then the isotherm should be a series of square. So we draw the outside square of the circles existing in every picture, and calculate standard deviation of temperature on the edge of square, it would precisely indicate the extent to which heat is evenly distributed. However, this method is not applied for our technical restriction.

6 Strengths and weaknesses

6.1 Strengths

We provide a practical reference sheet for the customers to select different type of pans. It can help baker make quickly decision for different situation. Our "overlapped circles" model have a better approximation for polygons with respect to rectangular. We don't rely on the complicity of algorthm, but the simple simulation of daily life, daily habits.

6.2 Weaknesses

Our model is discrete and based on the current size of facilities. It won't help when redesign the shape of pans. This decides that our model won't have a broad fitting range.

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7 Advertising Sheet